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Rethinking Modal Gender in the Context of the Universe of Tn-Types: Definitions and Mathematical Models for Tonicity and Phonicity

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Abstract: An indisputable cornerstone of the Western music tradition, the dialectic opposition between the major and minor grammatical modal genders has always been present in the imagination of musicians and music theorists for centuries. Such dialectics of opposition is especially important in the context of nineteenth-century harmonic dualism, with its ideas of tonicity and phonicity. These concepts serve as the main foundation for the way harmonic dualism conceives the major and minor worlds: two worlds with equivalent rights and properties, but with opposed polarities. This paper presents a redefinition of the terms tonicity and phonicity, translating those concepts to the context of post-tonal music theory. The terminologies of generatrix, tonicity, root, phonicity, vertex, and azimuth are explained in this paper, followed by propositions of mathematical models for those concepts, which spring from Richard Parncutt's root-salience model for pitch-class sets. In order to demonstrate the possibilities of using modal gender as a criterion for the study and classification of the universe of Tn-types, we will present a taxonomy of the 351 transpositional set types, which comprises the categories of tonic (major), phonic (minor) and neutral (genderless). In addition, there will be a small discussion on the effect of set symmetries and set asymmetries on the tonic/phonic properties of a Tn-type.

Keywords: Harmonic dualism; tonicity; phonicity; major/minor modal gender; post-tonal music theory.

Repensando os gêneros modais no contexto do universo de tipos transposicionais: definições e modelos matemáticos para tonicidade e fonicidade.

Resumo: Inegável pedra fundamental da tradição musical ocidental, a oposição dialética entre os gêneros musicais modais maior e menor tem se mantido continuamente presente na imaginação de músicos e teóricos musicais há séculos. Esta dialética de oposição é especialmente importante no contexto do dualismo harmônico do século XIX, com as suas ideias de tonicidade e fonicidade, que servem como embasamento à sua concepção dos mundos maior e menor: dois mundos com direitos e propriedades equivalentes, porém portando polaridades opostas. Este artigo apresenta uma redefinição dos termos tonicidade e fonicidade, transportando estes conceitos para o contexto da teoria musical pós-tonal. São apresentadas definições e explicações das terminologias geratriz, tonicidade, raiz, fonicidade, vértice e azimute, seguidas por proposições de modelos matemáticos para estes conceitos, que foram criados a partir do modelo de Richard Parncutt para a mensuração da saliência das raízes de conjuntos de classes de altura. Para demonstrar as possibilidades de utilização do conceito de gênero modal como critério para o estudo e classificação do universo de tipos transposicionais, é apresentada uma taxonomia dos 351 tipos transposicionais, classificados segundo as categorias tônico (maior), fônico (menor) e neutro (sem gênero), incluindo ainda uma pequena discussão sobre o efeito de simetrias e assimetrias intervalares nas propriedades tônico-fônicas de um tipo transposicional.

Palavras-chave: Dualismo harmônico. Tonicidade. Fonicidade. Gêneros modais maior/menor. Teoria da música pós-tonal.

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The contraposition of the modal grammatical gender categories of major and minor is a concept that has always been present in the imagination of musicians and music theorists for centuries: a dialectics of interplay between two modal worlds with equivalent rights and properties, but with opposed polarities (cf. LEVY, 1985: 11-17). Such duality of musical conception is true especially since the advent of polyphony, the practices of which eventually established the concept of the perfect triad – both in its major and minor versions – as the main contextualizing contrapuntal/harmonic element for composition[1]. It would hardly be far-fetched to state that the musical concept of modal gender is an indisputable cornerstone of the Western music tradition, since it has never ceased to play a pivotal role in defining the compositional properties of musical structures even in supposedly all-inclusive post-tonal theories from the twentieth century such as Edmond Costère's studies on the "*lois et styles des harmonies musicales*" (COSTÈRE, 1954).

Present at least since Zarlino's overt sixteenth-century duality between the division of the fifth at the harmonic mean of its string length or at its arithmetic mean, which creates, respectively, the major and minor triads (cf. WIENPAHL, 1959: 35), the opposition between major and minor is most notably the centerpiece of harmonic dualism, a body of theories crucial for the understanding of nineteenth-century tonality (cf. HARRISON, 1994) and whose most important figures were the theorists Moritz Hauptmann (1792-1868), Arthur von Oettingen (1836-1920), and Hugo Riemann (1849-1919). From these, Oettingen can be singled out for his influential theoretical contribution of the concepts of tonicity, phonicity, tonic root and phonic overtone (OETTINGEN, 1866: 27-35), which helped to establish a dualist origin for the perfect triads in a reasonable and logical way without resorting to a fictitious undertone series, as Hugo Riemann had once conjectured (cf. MICKELSEN, 1977: 14-17. KLUMPENHOUWER, 2002: 464-465). Also, Oettingen's concepts posit a major/minor gender marking to harmonic structures which is based on actual physical properties of sounds rather than on cultural theoretical constructs.

This paper presents an attempt to recast Oettingen's concepts of tonicity and phonicity, expanding and merging those ideas with new ones to form a broader theory that aims to provide a meaningful way to define and classify the elements within the universe of Tn-types[2] into two grammatical gender categories, major and minor, plus a third genderless neutral category.

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[1] Cf. Zarlino's sixteenth-century concept of *Harmonia Perfetta* in Wienpahl (1959: 37).

[2] In musical set-theory, a Tn-type (or transpositional type) is an equivalence class which represents the unique interval-class configuration shared by all the transpositions of a set of

Grounding principles for harmonic stability

Whenever a sonority is perceived to be a single unit with one unified definite pitch sensation and a harmonic timbre, we have what will be here defined as a "musical note". This unified pitch sensation phenomenon occurs when the sonority is comprised of harmonic pure-tone partials, that is, pure-tone partials whose frequency values belong to a single arithmetic progression (also known by musicians as a harmonic series), either perfectly or approximately (cf. PLOMP, 1967). The actual pitch sensation experienced will be directly related to a frequency value equal to the greatest common divisor of the frequencies of the partials of the sonority spectrum (partials which in this case become known as "harmonics"), even if that value does not physically occur in the sonority (PLOMP, 1967: 1526). It is this greatest common divisor frequency that is commonly called the fundamental tone (or first harmonic) of the note.

Since birth (or even before that) we are exposed to sounds with harmonic spectra, starting with the experience of the sounds of our own voices and of our parents' voices (cf. PARNCUTT, 1989: 48-56). The very process of learning the sounds of speech vowels, the sounds of nature, the sounds of vibrating strings and of columns of air in pipes develops in us a strong familiarity to sounds with harmonic spectra (or according to the definition here posited, to notes). The most audible and intensely experienced components of such note sounds are their first five harmonics (cf. PLACK; OXENHAM, 2005: 20-23), especially the ones not submitted to octave-related perceptual fusion: the first, third, and fifth harmonics; and to a much lesser degree, the seventh and ninth harmonics also imprint their marks on our experience. The higher the order of the harmonic, the harder it is to single it out from the timbral mass, as we have a somewhat impaired experience of that harmonic (PARNCUTT, 1988: 70).

When two notes are combined into a musical interval, all their individual pure-tone harmonic partials interact with each other. When these partials are very close by in frequency, they produce amplitude beatings that, when fast enough, create in our hearing mechanism the rather coarse undulating sensation usually known by theorists as roughness (cf. PARNCUTT, 1989: 25, 58-59. HELMHOLTZ, 1875: 278). This phenomenon of roughness has been explained through research both on the physiology of the inner ear and on the cognitive mechanisms involved in the processing of the perceived stimuli (cf. ZWICKER; FASTL, 2007: 23-60, 257-

different pitch-classes (cf. RAHN, 1980: 74-77). In the 12-tone equal-temperament universe, there are exactly 351 of such different interval-class configurations (COSTÈRE, 1954: 62).

264). Modern scholarship is still evaluating the strong role and contribution of roughness in the construction of the cultural percepts of musical consonance and dissonance (cf. TENNEY, 1960). With all this in mind, the resulting sonority of every musical interval will possess a certain amount of roughness (or rather, sensory dissonance), depending on how close by in frequency are the partials of the individual notes of that interval. The more notes combine at musical intervals matching the lowest intervals of a harmonic series, the more coincident in frequency will be the partials of their notes, and, because of this, the less roughness (or sensorial dissonance) that interval will possess (cf. BENSON, 2006: 151-153). Considering, as mentioned earlier, the ease of hearing the lower harmonics of a note and the ubiquity of our exposure to sounds with harmonic spectra, if we hereby define the intelligibility of a certain thing to be the measurement of the amount of our prior exposure to it (in the sense that the more we are exposed to that certain thing during our lifetime, the more familiar and therefore intelligible it will seem to us), then the lower the order of a harmonic partial, the more intelligible that partial is to us. These important harmonic-series relationships, by the principle of octave equivalence[3], can be abstracted to simple music intervals comprised of notes up to one octave apart, which by inference end up inheriting the intelligibility of their harmonic-series generating counterpart. These simple music intervals, which can be represented by the ratios between the frequencies of the fundamental tones of their two notes, are therefore the best models of harmonic intelligibility and, as we have seen earlier, of sensory consonance. We will then define here as primeval intervals the extremely intelligible and sensorially consonant intervals derived from the relationships found between the fundamental tone and the first ten harmonic partials of a note.

Thus, the primeval intervals are the five extremely intelligible simple intervals derived, as just explained, from the first, third, fifth, seventh, and ninth harmonics of a note. These intervals are, in order of decreasing intelligibility: the *diapason*[4] (ratio 2:1), the abstraction of the first, second, fourth, and eighth

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[3] The acceptance of the principle of octave equivalence, which is a very important theoretical component in this theory, would no doubt merit a long theoretical discussion of its own. For our purposes here, it should suffice to state that the octave equivalence is a concept that has been important to music theory for centuries (cf. HELMHOLTZ, 1875: 390-391). In the 6th century, Boethius already used the Latin term *aequisoni* (cf. GAFFURIUS, 1968: 118) to refer to this important relationship of equivalence, which is intuitively perceived to exist between notes one or more octaves apart.

[4] These classical names for music intervals, which are derived from ancient Greek (*diapason*, *diapente*) and Latin sources (*sesquiquarta*, *supertripartiens-quarta*, *sesquioctava*), have been used

harmonics, represented in the 12-tone equal-temperament realm by the perfect octave; the *diapente* (ratio 3:2), the abstraction of the third and sixth harmonics, represented in the 12-tone equal-temperament realm by the perfect fifth; the *sesquiquarta* (ratio 5:4), the abstraction of the fifth and tenth harmonics, represented in equal-temperament by the major third; the *supertripartiens-quarta* (ratio 7:4), the abstraction of the seventh harmonic, represented in equal-temperament rather coarsely by the minor seventh; and the *sesquioctava* (ratio 9:8), the abstraction of the ninth harmonic, represented in equal-temperament by the major second. From the differences and combinations of these primeval intervals come what will be called here the secondary intervals: the 3-limit perfect fourth (4:3), the 5-limit just-intonation minor second (16:15), the 5-limit minor third (6:5), the 5-limit minor sixth (8:5), the 5-limit major sixth (5:3), the 5-limit major seventh (15:8), and the tritone (several ratios such as 7:5).

Some clarification is needed to explain the application of the 12-tone equal-temperament system for matters relating to the harmonic series, which would naturally call for just-intonation. Surely, the just-intonation simple-ratio versions of the primeval intervals are the ones actually derived from the first ten partials of the harmonic spectrum. Nonetheless, the deviations of the equal-tempered intervals from their just-intonation counterparts are rather small: the perfect octave is exactly equal to the *diapason*, the perfect fifth is 2 cents[5] smaller than the *diapente*, the major third is 14 cents bigger than the *sesquiquarta*, the minor seventh is 31 cents bigger than the *supertripartiens-quarta*, and the major second is 4 cents smaller that the *sesquioctava*. Experimental studies have found that small inharmonicities generated by individual variations in tuning of the partials of as much as 3% in frequency (which is about 50 cents) are indeed well tolerated by our perception (cf. MOORE; PETERS; GLASBERG, 1985: 1866). Considering this and the usual, well-documented slight inharmonicity of the natural timbre of musical instruments (such as the piano, for example, as seen in SCHUCK; YOUNG, 1943), plus the everyday fact that even excellent musical performances involve some amount of mistuning and/or pitch vibrato, it is not surprising that even the biggest departure from the just-intonation model found (which is the minor seventh with its surplus of 31 cents) can still

by music theorists for hundreds of years to name the just-intonation intervals (cf. 18th-century HAWKINS, 1963: v. I, 116). The intention behind the use of these more classically-oriented names for intervals is to provide a means of formal differentiation between the just-intonation intervals and their 12-tone equal-temperament avatars.

5 A cent is a musical interval equal to one hundreth of a 12-tone equal-tempered minor second. In this sense, a perfect octave (*diapason*) is comprised of 1200 cents.

somewhat serve as a representative of the simple interval generated by the seventh harmonic, the *supertripartiens-quarta* (7:4). It seems that our perception has adapted quite well to the natural sound environment, learning to relate a certain range of intervals of very similar size to one basic just-intonation interval pattern. This knowledge, even if apprehended rather intuitively throughout history, is the grounding basis for all temperament methods (cf. BARBOUR, 1951, 1-13), a concept already present in the minds of early theorists such as fifteenth-century Gaffurius: "a fifth can be diminished by a very small, hidden and somewhat indefinite amount (as organists assert), which they call *participata*" (GAFFURIUS, 1968: 125). It is in this capacity that an equal-tempered interval can act as an avatar, a representative of a just interval, although with some amount of a consciously-perceived increase in sensory dissonance, or roughness. Thus, it is possible to consider the twelve-tone equal temperament system as an interesting practical compromise for formulating archetypical harmonic properties of sound combinations, since that temperament is able to provide a system that includes one, and only one, fixed-size avatar for every primeval and secondary interval. These theoretical archetypical harmonic properties would then be somewhat extended by inference to all their microtonal variations.

Generating harmonic structures from primeval intervals

Considering that primeval intervals are the basic models of harmonic stability (or rather, lack of accrued roughness, or sensory dissonance), we could use this knowledge to construct groups of notes of high harmonic stability by projecting a certain number of primeval intervallic relationships from a single point of departure either upwards (higher in the pitch scale) or downwards (lower in the pitch scale). In geometry, a generatrix is a "point, line, or surface whose motion generates a line, surface, or solid", according to The Merriam-Webster Online Dictionary (GENERATRIX, 2016). In an analogous manner, I borrow this term to call this harmonic point of departure a generatrix. Thus, this generatrix is the mediating element that binds together and grants cohesion to sonorities comprised of more than two notes. If we construct a group of notes projecting primeval intervals upwards from a generatrix, the notes of the group will be bound by a common fundamental, or root; if we construct a group of notes projecting primeval intervals downwards from a generatrix, the notes of the group will be bound by a common overtone. These are the basic ideas behind Oettingen's concept of tonicity and phonicity, respectively (OETTINGEN, 1866: 27-35): tonicity is "the property of an interval or chord to be grasped as a partial of a fundamental" (OETTINGEN apud KLUMPENHOUWER, 2002: 464), and phonicity is "the property of the pitches that constitute an interval or chord to possess common partials" (ibid). A tonic or phonic version of the same amalgam of primeval intervals should yield note groupings of a somewhat comparable and similar roughness (for they include the same intervals), although the tonic version will appear to be more fused (and therefore more stable), this because all notes of the group are related to partials of the generatrix note[6].

Historically, the construction of primeval consonant intervals from a generatrix point has been the preferred nineteenth-century dualist explanation for the source of the equivalence and excellence of the consonances of the major and minor triads (REHDING, 2003: 15), and this explanation has served as grounds for the importance bestowed to these two landmark harmonic structures. Fig. 1 shows an illustration of the concept of generatrix, with the pitch C being used as the point of departure (or rather, the generatrix) to construct, on the left, a major triad by upward motion of the primeval consonant intervals *diapente* (3:2) and *sesquiquarta* (5:4), and on the right, a minor triad by downward motion of the very same intervals.

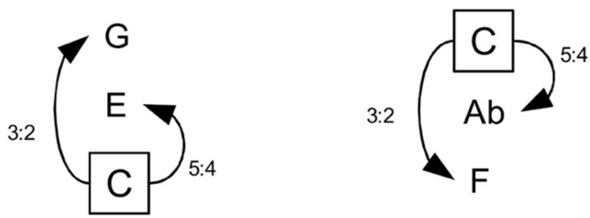

Fig 1: Illustration of the concept of generatrix.

In a way, we are just restating and amplifying centenary theoretical premises regarding the definition of the main formative elements of harmony, which could be summarized as in the following eminent historical statements: "there are three intervals directly intelligible: the octave, the fifth, the third (major). They are unchangeable" (HAUPTMANN, 1888: 5); all other intervals are to be explained "as the results of multiplication and involution of these three" (RIEMANN, 1903: 6); and "from the different positions of the Third, which is placed in counterpoint between the extremes of the Fifth or placed above the Octave, is born the variety of the Harmony" (ZARLINO apud WIENPAHL, 1959: 28).

[6] This fusing property has been called "tonalness" by Terhardt, which is "the degree to which a sound has the sensory properties of a single complex tone such as a speech vowel" (TERHARDT apud PARNCUTT, 1989: 25).

Tonicity and the tonic root

In the context of a sonority made out of several individual notes, a root is here defined as the note whose spectrum contains partials that coincide directly or in an octave-related manner to the great majority of the important partials of the collective spectrum of the sonority. Because of this, this note stands out perceptually as the overall center of the sonority. The more we have partials of the collective spectrum coinciding directly or in an octave-related manner with the partials of a root, the stronger this root is. In other words, the root is the note whose fundamental tone is related by unison or octave to the best candidate for a common fundamental tone for the entire sonority.

Tonicity is then hereby defined as the measure of the extent to which a sonority seems to spring from this common fundamental tone: the root, or tonic root; it is the extent to which the sonority contains pitch-classes whose overtones are bound together by a common fundamental tone. The more we have a strong and unique root for a sonority, the more tonicity this sonority bears. In this sense, the tonicity can be objectively quantified using Parncutt and Terhardt's method of root estimation and Parncutt's table of root-saliences (PARNCUTT, 1988: 74-76; 1997: 186-187; 2009: 127-132), as will be seen later. Fig. 2 shows an illustration of the concept of tonicity and tonic root: on the left, one sees a sonority with high tonicity; on the right, one sees a demonstration showing the coincidence of several important harmonics of the three upper notes of the sonority with harmonics of the bottom note, which is, therefore, the tonic root of the sonority.

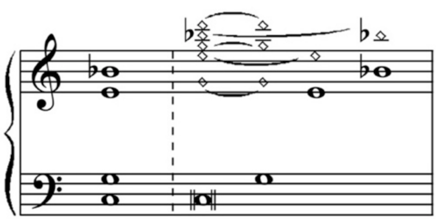

Fig 2: Illustration of the concept of tonicity and tonic root.

Phonicity and the phonic vertex

In the context of a sonority comprised of several individual notes, a vertex (a new term of my invention denoting Oettingen's "phonic overtone") is here defined as a pitch location where partials of the different constituent notes of the sonority intersect either directly or in an octave-related manner. It is a strong point of coincidence between the several different constituent spectra. In other words, the vertex is the note whose fundamental tone is related by unison or octave to the best candidate for a common overtone for the notes of the sonority. The more we have the different spectra of the sonority notes coinciding at a vertex either directly or in an octave-related manner, the stronger this vertex is.

Phonicity is then hereby defined as the measure of the extent to which a sonority produces this common overtone: the vertex, or phonic vertex; it is the extent to which the sonority contains pitch-classes that relate to different fundamental tones bound by a common overtone. The more we have a strong and unique vertex for a sonority, the more phonicity this sonority bears. The phonicity can be objectively quantified from the table of vertex-saliences, which will be here defined as an upside-down version of Parncutt's table of root-saliences, as will be seen later. Fig. 3 shows an illustration of the concept of phonicity and phonic vertex: on the left, one sees a sonority with high phonicity; on the right, one sees a demonstration showing the coincidence of harmonics of the three bottom notes of the sonority with an octave-related overtone of the fundamental of the top note, which is, therefore, the phonic vertex of the sonority.

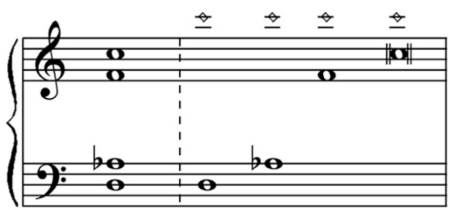

Fig 3: Illustration of the concept of phonicity and phonic vertex.

Azimuth

In its usual context, azimuth is a measurement for horizontal distance widely used in astronomy, artillery and navigation; it is "the horizontal direction expressed as the angular distance between the direction of a fixed point (as the observer's heading) and the direction of the object", according to The Merriam-Webster Online Dictionary (AZIMUTH, 2016). Thus, the usual four cardinal points can be represented with the azimuth values of 0° (North), 180° (South), 90° (East), and 270° or -90° (West), for example.

Adapting this idea for music theory purposes, the azimuth of a sonority is hereby defined as the sonority's bias towards a tonic or phonic nature. A sonority is considered to be tonic in nature when its roots are stronger than its vertices, and vice versa: a sonority is considered to be phonic in nature when its vertices are stronger than its roots. As a convention, this bias is here represented by an eastbound azimuth value with a positive sign for a sonority with tonic nature, and it is represented by a westbound azimuth value with a negative sign for a sonority with phonic nature. As for sonorities in which the tonicity equals the phonicity, these will be said to have a null azimuth (0°). In this way, we can, as a convention, represent the greatest tonic azimuth possible with the value +90° (completely eastbound), and the greatest phonic azimuth possible with the value -90° (completely westbound).

According to these given definitions, the quantification of the azimuth of a sonority requires that the methods for the quantification of tonicity and phonicity be based on similar measurement principles, but implemented in a symmetrically opposed manner. Thus, the azimuth will be calculated by simply subtracting the tonicity from the phonicity and normalizing the result so it fits into the desired range in degrees (-90° to +90°).

Richard Parncutt's root-salience model for pitch-class sets

According to the definition by Richard Parncutt (2009: 127), the salience of a pitch-class is the probability that a listener will experience a sensation for that pitch-class during the existence of a sonority. In a similar way, he defines the root-salience of a pitch-class as the measurement of the probability of this pitch-class being felt as the root of a sonority. A simple method of measuring root-salience has been experimentally devised by Parncutt based on a model by Ernst Terhardt (cf. PARNCUTT, 1988, 1989, 1997, 2009), which in its turn is based on the way our minds perceive sounds with harmonic spectra (or rather, notes). We are here adapting Parncutt's simplified 1997 root-salience method of calculation as a component for the measurement of the tonicity of a Tn-type.

The method of root estimation by Parncutt and Terhardt is essentially a process of sub-harmonic matching. If we take a sonority comprised of several notes, each partial of its collective spectrum will be considered by our minds to be a harmonic partial of a possible fundamental root tone in five different ways: as a first, third, fifth, seventh, and ninth harmonic. Each root tone possibility will have a probability weight proportional to the hierarchical strength (in the sense of degree of intelligibility) of the harmonic relationship it holds with the analyzed partial. Based on the analysis of the average audibility of harmonics in speech vowels (PARNCUTT, 1988: 73-76), the hierarchical strength for each one of those five harmonic relationships has been quantified by Parncutt in a tentative albeit simple and convenient approximate manner as follows: 10 probability points for a first-harmonic relationship, 5 points for a third-harmonic relationship, 3 points for a fifth-harmonic relationship, 2 points for a seventh-harmonic relationship, and 1 point for a ninth-harmonic relationship (PARNCUTT, 1997: 186-187). These intervallic relationships are called by Parncutt root supports. Each possible root tone will transfer its accrued probability weight to its pitch-class representative as root-salience points. After all partials of the collective spectrum are evaluated in this way, the pitch-class with the greatest accrued root-salience will be considered the perceptual root of that sonority.

Parncutt and Terhardt also considered that for a given sonority only the fundamental tones of each of its constituent notes would have some active and determinant effect on root formation (cf. PARNCUTT, 1988: 70-75). This makes sense if one considers that, taking individually each note component of the sonority, the root-defining effects of their higher overtones are negligible and redundant for two reasons: (a) if a fundamental tone of a note component can be seen as a harmonic of a possible root, so can its higher harmonics, which will only slightly reinforce the root-defining power already present in that fundamental; and (b) if a high overtone of a note component indeed points to a root that is different from its own fundamental tone, the root-defining power of that high harmonic towards that root will be masked by the presence of the partials of any other different note component whose fundamental tone actually points towards that same root. In both cases, the root-defining effect of the higher harmonics of each note component is somewhat diluted and accounted for in the collective effect of their own fundamentals. This simplifies considerably the root estimation process of a sonority, for instead of putting every single partial of its collective spectrum through the process of sub-harmonic matching, we can analyze only the effects of the fundamental tones of its constituent notes, and these fundamental tones can be further abstracted to the very pitch-classes that represent each note component. This simplification allowed Parncutt to use his calculation method to determine in a very useful manner the perceptual root of a theoretical abstraction such as a pitch-class set (cf. PARNCUTT, 2009).

With this simplification, the root-salience calculation method can be easily described as follows: each pitch-class of the sonority will grant: (a) 10 units of root-salience to the pitch-class an octave below that partial; (b) 5 units of root-salience to the pitch-class a perfect fifth below that partial; (c) 3 units of root-salience to the pitch-class a major third below that partial; (d) 2 units of root-salience to the pitch-class a minor seventh below that partial; and (e) 1 unit of root-salience to the pitch-class a major second below that partial. Another way of looking at this process is to consider that since we are looking for the possible roots of a pitch-class collection, we are analyzing its tonicity. We will be then searching for possible generatrices from where primeval intervals were constructed by upward motion. Considering that each primeval interval inherits the perceptual weight of its generating harmonic partial, we can take each pitch-class of the collection and assign the corresponding perceptual weights to the pitch-classes that rest downwards at the distances of the five primeval intervals. These calculations can be mathematically formalized by the following equation (Eq. 1):

$$rS_{(p)} = 10 \cdot N_{(p)} + 5 \cdot N_{(p+7)} + 3 \cdot N_{(p+4)} + 2 \cdot N_{(p+10)} + 1 \cdot N_{(p+2)}$$

where: $rS(p)$ = root-salience of pitch-class p ;
$N(p)$ = number of occurrences of pitch-class p in the sonority;

OBS: increments to p are to be calculated in arithmetic modulo 12.

Equation 1

For example, to find the perceptual root of the pitch-class collection {9 1 4}, we consider that the pitch-class 9 will give 10 root-salience points to pitch-class 9 (its octave), 5 points to pitch-class 2 (its perfect fifth below), 3 points to pitch-class 5 (its major third below), 2 points to pitch-class 11 (its minor seventh below), and 1 point to pitch-class 7 (its major second below). The same process is applied to the other pitch-classes of the set, 1 and 4. We then add all the root-salience points given for each one of the twelve pitch-classes, and the pitch-class with the largest amount of points is considered to be the root of the collection, as shown in Fig. 4.

	0	1	2	3	4	5	6	7	8	9	10	11
9			5			3		1		10		2
1		10		2			5			3		1
4	3		1		10		2			5		
total:	3	10	6	2	10	3	7	1	0	***18***	0	3

Fig 4: Root estimation for pitch-class set {9 1 4}.

In this example, pitch-class 9 clearly acts as the perceptual root of the pitch-class collection {9 1 4}, with 18 root-salience points. Since pitch-class 9 receives points from every single pitch-class present in the collection, all of these should rest upwards at primeval interval distances from the root; therefore, this collection contains a generatrix (and one with a tonic nature), which is coincident to its very root (pitch-class 9).

Parncutt (1997: 191-192) has argued that a very interesting use for this root-estimation method is its capability to correctly predict the root of a minor triad, giving a very reasonable explanation for one famous historical music theory mystery – the controversy about the origin and nature of the minor triad and the identity of its root (cf. LEVY, 1985: 11-17) – as shown in Fig. 5:

	0	1	2	3	4	5	6	7	8	9	10	11
0	10		2			5			3		1	
3		1		10		2			5			3
7	5			3		1		10		2		
total:	***15***	1	2	13	0	8	0	10	8	2	1	3

Fig 5: Root estimation for pitch-class set {0 3 7}.

Observing the root-salience data on the set {0 3 7}, we see that pitch-class 0 is clearly its root, although it received points from only two constituent pitch-classes; because of this fact, the root is not a generatrix. There is indeed one pitch-class, 5, which received points from all constituent pitch-classes, but since it does not belong to the collection, it also cannot be a tonic generatrix, because if a simultaneous interval is to exist, its two defining pitches must be sounding; thus, the generatrix must inevitably exist in the sonority. As a matter of fact, this collection actually has a phonic generatrix, the perfect fifth of the root (pitch-class 7), which is also the vertex of the set.

It is this formal separation between the notions of generatrix, root, and vertex that makes possible a theoretical reconciliation between 19th-century dualist theories and common historical musical practice[7], completely obliterating any need to posit the hypothetical "undertone series" unicorn. Thus separated, these concepts allow the minor triad to be theoretically conceived in a phonic manner, while keeping the historical and commonplace notion that its root is its lower note. In this way, what was considered by theorists such as Riemann and Oettingen (cf. RIEMANN, 1903: 10-11. RIEMANN, 1977: 219) to be the root of the minor triad (namely, the fifth of the bottom note, which is an idea that completely contradicts basic common notions found in musical practice) can retain its conceptual importance as a vertex and, more importantly, as a generatrix, while conceding the post of root to the usual traditional candidate: the bottom tone of the triad.

A vertex-salience model for pitch-class sets

In the same spirit as the prior given definitions, I am here defining the vertex-salience of a pitch-class as the measurement of its probability of being felt as the vertex of a sonority. It is possible to elaborate a measurement method for this property based on Parncutt's root-salience calculation method we just described earlier. Tonicity and phonicity are relationships created by actions of similar nature (because both arise from a process of projecting primeval intervals from a generatrix point in one single direction) but performed in opposite directions. Tonic relationships arise from upward (higher in pitch) projections from a generatrix, and phonic relationships arise from downward (lower in pitch) projections. Thus, we can conceive phonicity as an upside-down version of tonicity.

The method of vertex estimation then becomes essentially a process of harmonic matching. If we take a sonority comprised of several notes, each one of its constituent pitch-class representatives can be interpreted as being a fundamental tone of five different possibilities of harmonic partials: first, third, fifth, seventh, and ninth harmonics. Each vertex tone possibility will have a probability weight

[7] An early attempt at this reconciliation is Ernst Levy's concept of "telluric gravity" (cf. LEVY, 1985: 15).

proportional to the hierarchical strength (again in the sense of degree of intelligibility) of the harmonic relationship it holds with the analyzed pitch-class, just like in Parncutt's root-salience calculation method. Each possible vertex tone will transfer its accrued probability weight to its pitch-class representative as vertex-salience points. After all pitch-classes of the collective spectrum are evaluated in this way, the pitch-class with the greatest accrued vertex-salience will be considered the perceptual vertex of that sonority.

Following this reasoning, the vertex-salience calculation method can be easily described as follows: each pitch-class of the sonority will grant: (a) 10 units of vertex-salience to the pitch-class an octave above that pitch-class; (b) 5 units of vertex-salience to the pitch-class a perfect fifth above that pitch-class; (c) 3 units of vertex-salience to the pitch-class a major third above that pitch-class; (d) 2 units of vertex-salience to the pitch-class a minor seventh above that pitch-class; and (e) 1 unit of vertex-salience to the pitch-class a major second above that pitch-class. Another way of looking at this process is to consider that since we are looking for the possible vertices of a pitch-class collection, we are analyzing its phonicity. We will be then searching for possible generatrices from where primeval intervals were constructed by downward motion. Considering that each primeval interval inherits the perceptual weight of its generating harmonic partial, we can take each pitch-class of the collection and assign the corresponding perceptual weights to the pitch-classes that rest upwards at the distances of the five primeval intervals. These calculations can be mathematically formalized by the following equation (Eq. 2):

$$vS_{(p)} = 10 \cdot N_{(p)} + 5 \cdot N_{(p-7)} + 3 \cdot N_{(p-4)} + 2 \cdot N_{(p-10)} + 1 \cdot N_{(p-2)}$$

where: $vS(p)$ = vertex-salience of pitch-class p ;
$N(p)$ = number of occurrences of pitch-class p in the sonority;

OBS: increments to p are to be calculated in arithmetic modulo 12.

Equation 2

For example, to find the perceptual vertex of the pitch-class collection {9 1 4}, we consider that the pitch-class 9 will give 10 root-salience points to pitch-class 9 (its octave), 5 points to pitch-class 4 (its perfect fifth above), 3 points to pitch-class 1 (its major third above), 2 points to pitch-class 7 (its minor seventh above), and 1 point to pitch-class 11 (its major second above). The same process is applied to the other pitch-classes of the set, 1 and 4. We then add all the vertex-salience points given for each one of the twelve pitch-classes, and the pitch-class with the largest amount of points is considered to be the vertex of the collection, as shown in Fig. 6.

	0	1	2	3	4	5	6	7	8	9	10	11
9		3			5			2		10		1
1		10		1		3			5			2
4			2		10		1		3			5
total:	0	13	2	1	***15***	3	1	2	8	10	0	8

Fig 6: Vertex estimation for pitch-class set {9 1 4}.

In this example, pitch-class 4 clearly acts as the perceptual vertex of the pitch-class collection {9 1 4}, with 15 vertex-salience points. The vertex received points from only two constituent pitch-classes, which means it is not a generatrix. There is indeed one pitch-class, 11, which received points from all constituent pitch-classes, but since it does not belong to the collection, it also cannot be a phonic generatrix, for the same reasons already discussed earlier. As we have seen before, this set has a generatrix of tonic nature that is also its root: pitch-class 9.

Calculation methods for tonicity, phonicity, and azimuth

The calculation of the tonicity of a pitch-class set can be generated from its table of root-saliences. In the model I propose here, the tonicity value is set to be dependent on the relative strength of the root, on the amount of segregation of the biggest root-salience from the root-saliences of the other constituent pitch-classes, and on its uniqueness. Thus, the following assumptions are considered: (a) the tonicity is directly proportional to the fraction of the total root-salience points generated by the set that was concentrated on the root; (b) the tonicity is directly proportional to the amount a root surpasses the second greatest root-salience value (here it is possible that these values be the same, if the set has multiple roots); and (c) the tonicity is inversely proportional to the number of roots in the set. These assumptions implement those considerations of root strength, segregation and uniqueness, respectively. Thus, a preliminary calculation for the tonicity of a set can be obtained in the following manner:

$$preTon = \frac{St_{rt} \cdot Sg_{rt}}{N_{rt}}$$

where: St_{rt} = root-strength factor;
Sg_{rt} = root-segregation factor;
N_{rt} = number of roots in the set;

and:

$$St_{rt} = \frac{rS_{(max)}}{\sum_{i=0}^{11} rS_{(i)}}$$

$$Sg_{rt} = \frac{rS_{(max)}}{rS_{(second\ max)}}$$

where: $rS_{(max)}$ = greatest root-salience;
$rS_{(i)}$ = root-salience of pitch-class i;
$rS_{(second\ max)}$ = second greatest root-salience, even if it is equal to the greatest one (in the case of multiple roots);

Equation 3 (part 1)

We can then normalize (that is, scale to a reasonable and convenient measurement reading) the calculated preliminary tonicity, setting its maximum possible value to 1. This can be done by dividing the result by a constant *K*, which should be the largest possible value for a preliminary tonicity. After analysis of the calculated results for all 351 Tn-types, which was performed with the aid of a computer software of my own concoction, the constant *K* is actually the preliminary tonicity value for a set containing a single pitch-class: Tn-type (0). Finally, we convert the result into a percentage-style value multiplying it by 100, so that the maximum tonicity possible yields a value of 100% tonicity. The final equation (Eq. 3) is as follows:

$$Ton = \left(\frac{St_{rt} \cdot Sg_{rt}}{N_{rt}}\right) \cdot \frac{100}{K} \qquad \text{where:} \qquad K = \frac{\frac{10}{21} \cdot \frac{10}{5}}{1} = \frac{20}{21}$$

Equation 3 (part 2)

In the model I propose here, the calculation of the phonicity of a pitch-class is performed in the exact same way, but it is generated instead from the table of vertex-saliences of the set. The very same considerations and assumptions taken for the calculation of the tonicity will hold here: (a) the phonicity is directly proportional to the fraction of the total vertex-salience points generated by the set that was concentrated on the vertex; (b) the phonicity is directly proportional to the amount a vertex surpasses the second largest vertex-salience value (even if these values are the same, in the case of multiple vertices); and (c) the phonicity is inversely proportional to the number of vertices in the set. These assumptions implement, once again, the considerations of vertex strength, segregation and uniqueness, respectively. We can then calculate a preliminary value for phonicity:

$$prePhon = \frac{St_{vtx} \cdot Sg_{vtx}}{N_{vtx}}$$

where: St_{vtx} = vertex-strength factor;
Sg_{vtx} = vertex-segregation factor;
N_{vtx} = number of vertices in the set;

and:

$$St_{vtx} = \frac{vS_{(max)}}{\sum_{i=0}^{11} vS_{(i)}}$$

$$Sg_{vtx} = \frac{vS_{(max)}}{vS_{(second\,max)}}$$

where: $vS_{(max)}$ = greatest vertex-salience;

$vS_{(i)}$ = vertex-salience of pitch-class i ;

$vS_{(second\,max)}$ = second greatest vertex-salience, even if it is equal to the greatest one (in the case of multiple vertices);

Equation 4 (part 1)

We should then also normalize the calculated preliminary phonicity, setting its maximum possible value to 1, and we finish converting the result into a percentage-style value multiplying it by 100. This assures that the maximum phonicity possible will yield a value of 100% phonicity. Just as in the case of tonicity, the largest possible value for the normalizing constant *K* is the preliminary phonicity value for a set containing a single pitch-class: Tn-type (0). The final equation (Eq. 4) is as follows:

$$Phon = \left(\frac{St_{vtx} \cdot Sg_{vtx}}{N_{vtx}}\right) \cdot \frac{100}{K} \qquad \text{where:} \qquad K = \frac{\frac{10}{21} \cdot \frac{10}{5}}{1} = \frac{20}{21}$$

Equation 4 (part 2)

The calculation method for the azimuth of a pitch-class set is done simply by subtracting the set phonicity from the set tonicity, and then normalizing the value dividing it by a constant *ß*, so it ranges from -1 to +1. This normalization constant *ß* is the absolute value for the greatest possible difference between tonicity and phonicity, which occurs, after analysis of the calculated results for all 351 Tn-types performed with the aid of computer software, for the Tn-types (0 3 7) and (0 4 7), the minor and major triads, which yield the differences of -25.15385 and 25.15385, respectively. After dividing the difference by this constant *ß*, we multiply the value by 90, scaling it to an angular measurement ranging from -90° (the azimuth for a minor triad, represented by a completely westbound direction) to +90° (the azimuth for a major triad, represented by a completely eastbound direction). The resulting equation (Eq. 5) is the following:

$$Azi = (Ton - Phon) \cdot \frac{90}{\beta} \qquad \text{where:} \qquad \beta = 25.15385$$

Equation 5

Tonicity, phonicity, and azimuth as criteria for the classification of Tn-types

The concepts of tonicity and phonicity have served historically[8] to define the borders between the ideas of major and minor, respectively, applying a kind of grammatical gender marking to the triadic harmonic structures. In this respect, the azimuth concept, as a measurement of the major-minor bias of a Tn-type, can be used as a sorting agent and a grammatical gender measurement tool in the study of Tn-types and their musical properties.

From the 351 Tn-types possible in the twelve-tone equal-temperament universe, 95 are symmetrical sets[9] and 256 are asymmetrical sets (cf. COSTÈRE, 1954: 73). Due to their very symmetry, all symmetrical Tn-types yield identical values for tonicity and phonicity and therefore will bear zero-value null azimuths. These Tn-types can be said to be always genderless, or neutral sets. Table 1 shows the list of the 95 genderless neutral symmetrical Tn-types, in decreasing order of tonicity and phonicity.

Regarding the remaining 256 asymmetrical sets, these can be subdivided in 128 pairs consisting of a Tn-type and its inversion[10]. Due to their very inversional relationship, the tonicity value of a Tn-type will equal the phonicity value of its inversion, and vice versa. If a Tn-type has a bias towards a tonic nature (or better put, if it has a positive value azimuth), its inversion will have an equal bias towards a phonic nature, bearing the same absolute azimuth value of its pair, but with an opposite sign. From these 128 pairs of asymmetrical Tn-types, 98 pairs are composed by members with non-zero azimuth values, each bearing therefore a gender, either tonic (positive azimuth) or phonic (negative azimuth), whose tendency strength is revealed by its very azimuth value. These 98 pairs are comprised of one tonic (major) set and its corresponding phonic (minor) inverted twin set, both with symmetrically opposed equal azimuth values. Table 2 shows a list of the 98 pairs of azimuth-biased Tn-types, in decreasing order of absolute azimuth values.

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8 Cf. for example the contraposition between the arithmetic (minor) genus and the harmonic (major) genus in Tartini's 18th-century *Trattato di Musica Secondo la Vera Scienza dell'Armonia* (TARTINI, 1754: 66).

9 A pitch-class set is considered to be symmetrical when its interval-class configuration can be arranged in such a way as to have the same sequence of interval-classes occurring upwards and downwards a given pitch axis point (cf. COSTÈRE, 1954: 72).

10 It is useful here to remark that this idea of a pair of a Tn-type and its inversion represents the well-known musical set theory concept of a Tn/TnI-type (cf. RAHN, 1980: 76).

	TON	PHON	T_n-Type	AZI
1	100.0	100.0	(0)	0.0°
2	56.25	56.25	(0 5)	0.0°
3	42.25	42.25	(0 4)	0.0°
4	32.73	32.73	(0 2)	0.0°
5	31.15	31.15	(0 3 4 7)	0.0°
6	30.10	30.10	(0 2 3 5)	0.0°
7	29.40	29.40	(0 2 4 6 9)	0.0°
8	28.44	28.44	(0 2 7)	0.0°
9	27.00	27.00	(0 1 5 8)	0.0°
10	25.13	25.13	(0 2 4)	0.0°
11	24.50	24.50	(0 2 3 4 6 9)	0.0°
12	23.82	23.82	(0 3 5 8)	0.0°
13	22.86	22.86	(0 2 4 6)	0.0°
14	22.58	22.58	(0 2 5 7)	0.0°
15	22.22	22.22	(0 1 3 5 6 9)	0.0°
16	22.22	22.22	(0 1 2 3 6 9)	0.0°
17	21.82	21.82	(0 1 2)	0.0°
18	21.63	21.63	(0 1 4 5)	0.0°
19	21.60	21.60	(0 1 4 7 8)	0.0°
20	21.33	21.33	(0 1 2 7)	0.0°
21	20.83	20.83	(0 1 3 6 8 9)	0.0°
22	20.25	20.25	(0 1 3 4 8)	0.0°
23	20.09	20.09	(0 2 4 8)	0.0°
24	20.06	20.06	(0 2 4 7 9)	0.0°

	TON	PHON	T_n-Type	AZI
25	19.69	19.69	(0 1 2 4 6 8 10)	0.0°
26	19.61	19.61	(0 2 3 5 6 8)	0.0°
27	19.27	19.27	(0 1 3 5 6)	0.0°
28	19.06	19.06	(0 3 4 5 8)	0.0°
29	18.52	18.52	(0 1 4 6 7 9)	0.0°
30	18.52	18.52	(0 1 3 4 7 9)	0.0°
31	18.29	18.29	(0 2 3 4 6)	0.0°
32	17.60	17.60	(0 1 3 4)	0.0°
33	17.50	17.50	(0 1 2 4 5 6 9)	0.0°
34	17.20	17.20	(0 1 2 3 4 5)	0.0°
35	17.07	17.07	(0 1 2 6 8)	0.0°
36	16.88	16.88	(0 1 2 3 7 8)	0.0°
37	16.88	16.88	(0 1 2 3 4 8)	0.0°
38	16.81	16.81	(0 2 3 4 5 6 8)	0.0°
39	16.71	16.71	(0 2 3 4 5 7)	0.0°
40	16.71	16.71	(0 1 3 5 7 8)	0.0°
41	16.71	16.71	(0 1 2 5 7 9)	0.0°
42	16.58	16.58	(0 1 3 5 6 8 10)	0.0°
43	15.88	15.88	(0 1 3 4 6 7)	0.0°
44	15.87	15.87	(0 1 2 5 6 8 9)	0.0°
45	15.08	15.08	(0 1 2 3 4)	0.0°
46	15.05	15.05	(0 1 2 5 6 7)	0.0°
47	15.05	15.05	(0 1 2 4 5 6)	0.0°
48	15.04	15.04	(0 1 2 3 4 7 9)	0.0°

Table 1: List of the 95 genderless neutral symmetrical Tn-types.

	TON	PHON	T_n-Type	AZI
49	14.33	14.33	(0 1 3 4 5 7 8)	0.0°
50	14.33	14.33	(0 1 2 4 6 7 8)	0.0°
51	13.78	13.78	(0 1 2 4 5 7 9 10)	0.0°
52	13.78	13.78	(0 1 2 3 4 5 6 9)	0.0°
53	13.16	13.16	(0 1 2 4 5 7 8 9)	0.0°
54	13.16	13.16	(0 1 2 3 5 6 7 8)	0.0°
55	13.16	13.16	(0 1 2 3 4 7 8 9)	0.0°
56	13.16	13.16	(0 1 2 3 4 5 8 9)	0.0°
57	12.53	12.53	(0 1 2 3 4 5 6 7)	0.0°
58	12.50	12.50	(0 6)	0.0°
59	12.50	12.50	(0 3)	0.0°
60	12.50	12.50	(0 1)	0.0°
61	11.70	11.70	(0 1 2 3 4 5 6 7 8)	0.0°
62	9.38	9.38	(0 2 6 8)	0.0°
63	9.38	9.38	(0 1 6 7)	0.0°
64	9.38	9.38	(0 1 5 6)	0.0°
65	8.00	8.00	(0 2 4 6 8)	0.0°
66	7.92	7.92	(0 2 4 5 7 9)	0.0°
67	7.50	7.50	(0 1 3 4 6 8 10)	0.0°
68	7.50	7.50	(0 1 2 3)	0.0°
69	7.22	7.22	(0 4 8)	0.0°
70	6.67	6.67	(0 1 2 6 7 8)	0.0°
71	6.56	6.56	(0 2 3 4 5 6 7 9)	0.0°
72	6.56	6.56	(0 1 2 4 6 7 8 10)	0.0°

	TON	PHON	T_n-Type	AZI
73	6.56	6.56	(0 1 2 4 5 6 8 10)	0.0°
74	6.56	6.56	(0 1 2 3 5 7 8 10)	0.0°
75	6.56	6.56	(0 1 2 3 4 6 8 10)	0.0°
76	6.25	6.25	(0 1 3 4 5 6 8 9)	0.0°
77	6.25	6.25	(0 1 2 3 6 7 8 9)	0.0°
78	6.07	6.07	(0 1 2 3 4 5 6)	0.0°
79	5.83	5.83	(0 1 2 3 4 6 7 9 10)	0.0°
80	5.25	5.25	(0 1 2 3 4 5 6 7 8 9)	0.0°
81	5.00	5.00	(0 1 4 5 8 9)	0.0°
82	4.17	4.17	(0 3 6)	0.0°
83	3.89	3.89	(0 1 2 4 5 6 8 9 10)	0.0°
84	3.89	3.89	(0 1 2 3 5 6 7 8 10)	0.0°
85	3.89	3.89	(0 1 2 3 4 5 6 8 10)	0.0°
86	3.12	3.12	(0 1 3 4 6 7 9 10)	0.0°
87	2.62	2.62	(0 1 2 3 4 6 7 8 9 10)	0.0°
88	2.62	2.62	(0 1 2 3 4 5 7 8 9 10)	0.0°
89	2.62	2.62	(0 1 2 3 4 5 6 8 9 10)	0.0°
90	2.62	2.62	(0 1 2 3 4 5 6 7 9 10)	0.0°
91	2.22	2.22	(0 2 4 6 8 10)	0.0°
92	2.10	2.10	(0 1 2 3 4 5 6 7 8 10)	0.0°
93	1.56	1.56	(0 3 6 9)	0.0°
94	1.36	1.36	(0 1 2 3 4 5 6 7 8 9 10)	0.0°
95	0.73	0.73	(0 1 2 3 4 5 6 7 8 9 10 11)	0.0°

Table 1 (continued): List of the 95 genderless neutral symmetrical Tn-types.

	TON	PHO	Tn-Type	AZI	AZI	Tn-Type	TON	PHO
1	54.00	28.85	(0 4 7)	90.00°	-90.00°	(0 3 7)	28.85	54.00
2	33.75	9.38	(0 4 6 7)	87.21°	-87.21°	(0 1 3 7)	9.38	33.75
3	45.45	21.63	(0 3 6 8)	85.23°	-85.23°	(0 2 5 8)	21.63	45.45
4	26.67	5.00	(0 2 3 6 8)	77.52°	-77.52°	(0 2 5 6 8)	5.00	26.67
5	28.57	7.50	(0 3 4 6 8)	75.39°	-75.39°	(0 2 4 5 8)	7.50	28.57
6	36.36	17.31	(0 2 3 6 9)	68.18°	-68.18°	(0 1 3 6 9)	17.31	36.36
7	40.50	21.63	(0 1 4 7)	67.50°	-67.50°	(0 3 6 7)	21.63	40.50
8	27.79	9.38	(0 1 4 6)	65.88°	-65.88°	(0 2 5 6)	9.38	27.79
9	24.07	9.00	(0 2 3 4 7)	53.91°	-53.91°	(0 3 4 5 7)	9.00	24.07
10	22.23	7.50	(0 3 4 5 6)	52.71°	-52.71°	(0 1 2 3 6)	7.50	22.23
11	19.27	5.00	(0 1 4 5 6)	51.05°	-51.05°	(0 1 2 5 6)	5.00	19.27
12	20.83	6.67	(0 2 3 4 6 8)	50.69°	-50.69°	(0 2 4 5 6 8)	6.67	20.83
13	21.60	7.50	(0 2 3 4 8)	50.45°	-50.45°	(0 1 2 4 8)	7.50	21.60
14	19.06	5.00	(0 1 4 6 7)	50.30°	-50.30°	(0 1 3 6 7)	5.00	19.06
15	21.60	9.00	(0 1 4 5 8)	45.08°	-45.08°	(0 3 4 7 8)	9.00	21.60
16	20.06	7.50	(0 1 2 4 7 8)	44.92°	-44.92°	(0 1 4 6 7 8)	7.50	20.06
17	20.06	7.50	(0 1 2 3 4 7)	44.92°	-44.92°	(0 3 4 5 6 7)	7.50	20.06
18	43.79	31.25	(0 3 5)	44.86°	-44.86°	(0 2 5)	31.25	43.79
19	21.63	9.38	(0 4 5 6)	43.86°	-43.86°	(0 1 2 6)	9.38	21.63
20	15.04	3.21	(0 1 3 4 5 8 9)	42.30°	-42.30°	(0 1 2 4 5 8 9)	3.21	15.04
21	16.71	5.00	(0 1 3 4 5 8)	41.91°	-41.91°	(0 3 4 5 7 8)	5.00	16.71
22	18.80	7.50	(0 2 3 4 6 7)	40.44°	-40.44°	(0 1 3 4 5 7)	7.50	18.80
23	15.87	4.76	(0 1 3 4 6 7 9)	39.76°	-39.76°	(0 2 3 5 6 8 9)	4.76	15.87
24	20.06	9.00	(0 1 3 5 8)	39.56°	-39.56°	(0 3 5 7 8)	9.00	20.06
25	18.52	7.50	(0 1 2 5 6 9)	39.42°	-39.42°	(0 1 2 5 8 9)	7.50	18.52
26	17.86	7.14	(0 1 2 3 6 7 9)	38.34°	-38.34°	(0 1 2 3 6 8 9)	7.14	17.86
27	34.09	23.47	(0 2 6)	37.99°	-37.99°	(0 4 6)	23.47	34.09
28	17.54	7.50	(0 1 3 5 8 9)	35.94°	-35.94°	(0 1 4 6 8 9)	7.50	17.54
29	17.54	7.50	(0 1 3 5 6 8)	35.94°	-35.94°	(0 2 3 5 7 8)	7.50	17.54
30	14.33	4.29	(0 1 2 3 4 7 8)	35.92°	-35.92°	(0 1 4 5 6 7 8)	4.29	14.33
31	31.15	21.63	(0 3 4 8)	34.06°	-34.06°	(0 1 4 8)	21.63	31.15
32	32.84	23.44	(0 3 5 6)	33.65°	-33.65°	(0 1 3 6)	23.44	32.84
33	16.71	7.50	(0 2 3 4 7 8)	32.96°	-32.96°	(0 1 4 5 6 8)	7.50	16.71
34	16.06	7.08	(0 1 2 3 5 6)	32.10°	-32.10°	(0 1 3 4 5 6)	7.08	16.06
35	15.17	6.43	(0 1 2 3 4 6 7)	31.27°	-31.27°	(0 1 3 4 5 6 7)	6.43	15.17
36	15.87	7.14	(0 1 3 4 6 8 9)	31.24°	-31.24°	(0 1 3 5 6 8 9)	7.14	15.87
37	15.87	7.14	(0 1 3 4 5 6 9)	31.24°	-31.24°	(0 1 2 3 5 6 9)	7.14	15.87
38	15.04	6.43	(0 2 3 4 6 7 8)	30.80°	-30.80°	(0 1 2 4 5 6 8)	6.43	15.04

Table 2: List of the 98 pairs of azimuth-biased Tn-types.

	TON	PHO	T_n-Type	AZI	AZI	T_n-Type	TON	PHO
39	15.04	6.79	(0 1 2 4 5 7 9)	29.53°	-29.53°	(0 2 4 5 7 8 9)	6.79	15.04
40	25.57	17.60	(0 2 3 6)	28.50°	-28.50°	(0 3 4 6)	17.60	25.57
41	15.88	8.33	(0 1 3 4 6 9)	27.01°	-27.01°	(0 2 3 5 6 9)	8.33	15.88
42	13.16	5.94	(0 1 3 4 5 6 7 8)	25.83°	-25.83°	(0 1 2 3 4 5 7 8)	5.94	13.16
43	25.00	18.06	(0 1 3 6 8)	24.82°	-24.82°	(0 2 5 7 8)	18.06	25.00
44	27.79	21.63	(0 3 4 5)	22.02°	-22.02°	(0 1 2 5)	21.63	27.79
45	20.83	15.05	(0 1 2 3 6 8)	20.69°	-20.69°	(0 2 5 6 7 8)	15.05	20.83
46	25.80	20.09	(0 1 3 5)	20.45°	-20.45°	(0 2 4 5)	20.09	25.80
47	8.33	3.12	(0 2 3 6 8 9)	18.64°	-18.64°	(0 1 3 6 7 9)	3.12	8.33
48	26.67	21.60	(0 3 6 7 8)	18.13°	-18.13°	(0 1 2 5 8)	21.60	26.67
49	20.83	15.88	(0 1 3 6 7 8)	17.71°	-17.71°	(0 1 2 5 7 8)	15.88	20.83
50	20.64	16.07	(0 1 3 4 5)	16.36°	-16.36°	(0 1 2 4 5)	16.07	20.64
51	23.53	19.06	(0 3 5 6 8)	16.00°	-16.00°	(0 2 3 5 8)	19.06	23.53
52	17.86	13.61	(0 1 2 3 6 7 8)	15.18°	-15.18°	(0 1 2 5 6 7 8)	13.61	17.86
53	25.31	21.33	(0 4 5 7)	14.24°	-14.24°	(0 2 3 7)	21.33	25.31
54	20.83	16.88	(0 2 3 6 7 8)	14.16°	-14.16°	(0 1 2 5 6 8)	16.88	20.83
55	19.69	15.75	(0 1 3 5 6 7 9)	14.09°	-14.09°	(0 2 3 4 6 8 9)	15.75	19.69
56	19.61	15.88	(0 3 4 5 6 8)	13.33°	-13.33°	(0 2 3 4 5 8)	15.88	19.61
57	24.92	21.60	(0 3 4 6 7)	11.89°	-11.89°	(0 1 3 4 7)	21.60	24.92
58	20.25	17.07	(0 4 5 6 7)	11.39°	-11.39°	(0 1 2 3 7)	17.07	20.25
59	20.25	17.07	(0 1 4 5 7)	11.39°	-11.39°	(0 2 3 6 7)	17.07	20.25
60	22.22	19.06	(0 1 4 7 9)	11.32°	-11.32°	(0 1 4 6 9)	19.06	22.22
61	30.08	27.00	(0 2 4 7)	11.03°	-11.03°	(0 3 5 7)	27.00	30.08
62	22.23	19.27	(0 1 3 4 6)	10.61°	-10.61°	(0 2 3 5 6)	19.27	22.23
63	24.07	21.60	(0 2 4 7 8)	8.83°	-8.83°	(0 1 4 6 8)	21.60	24.07
64	24.07	21.60	(0 1 2 4 7)	8.83°	-8.83°	(0 3 5 6 7)	21.60	24.07
65	22.56	20.25	(0 2 4 6 7)	8.27°	-8.27°	(0 1 3 5 7)	20.25	22.56
66	21.24	19.06	(0 2 4 5 7)	7.79°	-7.79°	(0 2 3 5 7)	19.06	21.24
67	6.25	4.17	(0 1 3 4 6 7 8 9)	7.45°	-7.45°	(0 1 2 3 5 6 8 9)	4.17	6.25
68	18.80	16.88	(0 2 4 6 7 8)	6.90°	-6.90°	(0 1 2 4 6 8)	16.88	18.80
69	17.70	15.88	(0 2 4 5 6 7)	6.49°	-6.49°	(0 1 2 3 5 7)	15.88	17.70
70	17.70	15.88	(0 1 2 4 5 7)	6.49°	-6.49°	(0 2 3 5 6 7)	15.88	17.70
71	18.52	16.71	(0 3 5 6 7 8)	6.46°	-6.46°	(0 1 2 3 5 8)	16.71	18.52
72	17.50	15.75	(0 2 4 5 6 8 9)	6.26°	-6.26°	(0 1 3 4 5 7 9)	15.75	17.50
73	17.50	15.75	(0 2 3 5 7 8 9)	6.26°	-6.26°	(0 1 2 4 6 7 9)	15.75	17.50
74	17.50	15.75	(0 1 2 3 4 6 9)	6.26°	-6.26°	(0 2 3 4 5 6 9)	15.75	17.50
75	15.88	14.22	(0 1 4 5 6 7)	5.94°	-5.94°	(0 1 2 3 6 7)	14.22	15.88
76	15.17	13.61	(0 1 2 4 5 6 7)	5.56°	-5.56°	(0 1 2 3 5 6 7)	13.61	15.17

Table 2 (continued): List of the 98 pairs of azimuth-biased Tn-types.

	TON	PHO	T_n-Type	AZI	AZI	T_n-Type	TON	PHO
77	15.87	14.33	(0 3 4 5 6 7 8)	5.54°	-5.54°	(0 1 2 3 4 5 8)	14.33	15.87
78	15.87	14.33	(0 1 3 4 6 7 8)	5.54°	-5.54°	(0 1 2 4 5 7 8)	14.33	15.87
79	15.31	13.78	(0 1 2 4 5 6 8 9)	5.48°	-5.48°	(0 1 3 4 5 7 8 9)	13.78	15.31
80	20.42	19.34	(0 2 3 5 7 9)	3.84°	-3.84°	(0 2 4 6 7 9)	19.34	20.42
81	18.06	17.07	(0 1 2 4 6)	3.56°	-3.56°	(0 2 4 5 6)	17.07	18.06
82	17.50	16.58	(0 1 2 4 6 8 9)	3.30°	-3.30°	(0 1 3 5 7 8 9)	16.58	17.50
83	17.50	16.58	(0 1 2 3 5 7 9)	3.30°	-3.30°	(0 2 4 6 7 8 9)	16.58	17.50
84	15.87	15.04	(0 2 3 5 6 7 8)	2.99°	-2.99°	(0 1 2 3 5 6 8)	15.04	15.87
85	15.87	15.04	(0 1 4 6 7 8 9)	2.99°	-2.99°	(0 1 2 3 5 8 9)	15.04	15.87
86	17.54	16.71	(0 1 2 4 7 9)	2.97°	-2.97°	(0 2 3 4 7 9)	16.71	17.54
87	16.58	15.75	(0 2 3 4 6 7 9)	2.97°	-2.97°	(0 2 3 5 6 7 9)	15.75	16.58
88	18.52	17.70	(0 1 3 4 6 8)	2.94°	-2.94°	(0 2 4 5 7 8)	17.70	18.52
89	17.70	16.88	(0 1 2 4 6 7)	2.94°	-2.94°	(0 1 3 5 6 7)	16.88	17.70
90	14.51	13.78	(0 2 4 5 6 7 8 9)	2.60°	-2.60°	(0 1 2 3 4 5 7 9)	13.78	14.51
91	14.51	13.78	(0 2 3 4 5 7 8 9)	2.60°	-2.60°	(0 1 2 4 5 6 7 9)	13.78	14.51
92	14.51	13.78	(0 1 2 3 5 7 8 9)	2.60°	-2.60°	(0 1 2 4 6 7 8 9)	13.78	14.51
93	15.04	14.33	(0 1 3 5 6 7 8)	2.55°	-2.55°	(0 1 2 3 5 7 8)	14.33	15.04
94	15.04	14.33	(0 1 3 4 5 6 8)	2.55°	-2.55°	(0 2 3 4 5 7 8)	14.33	15.04
95	15.04	14.33	(0 1 2 4 7 8 9)	2.55°	-2.55°	(0 1 2 5 7 8 9)	14.33	15.04
96	15.87	15.17	(0 1 2 3 4 6 8)	2.52°	-2.52°	(0 2 4 5 6 7 8)	15.17	15.87
97	13.16	12.53	(0 1 2 3 4 6 7 8)	2.23°	-2.23°	(0 1 2 4 5 6 7 8)	12.53	13.16
98	18.52	18.00	(0 3 4 6 7 8)	1.86°	-1.86°	(0 1 2 4 5 8)	18.00	18.52

Table 2 (continued): List of the 98 pairs of azimuth-biased Tn-types.

The remaining 30 pairs of asymmetrical Tn-types are comprised of genderless neutral members yielding zero-value azimuths. These Tn-types are a mixture of a symmetrical Tn-type, which yields equal values for tonicity and phonicity, with other pitch-classes whose root-supporting and vertex-supporting effects are not big enough to upset the original root-and-vertex formation capability of their symmetrical accompanying group. Table 3 shows a list of the 30 pairs of asymmetrical azimuth-unbiased Tn-types, in decreasing order of tonicity and phonicity.

	TON	PHO	T_n-Type	AZI	AZI	T_n-Type	TON	PHO
1	37.50	37.50	(0 1 6)	0.00°	0.00°	(0 5 6)	37.50	37.50
2	28.85	28.85	(0 1 5)	0.00°	0.00°	(0 4 5)	28.85	28.85
3	28.17	28.17	(0 1 4)	0.00°	0.00°	(0 3 4)	28.17	28.17
4	22.97	22.97	(0 1 3 5 7 9)	0.00°	0.00°	(0 2 4 6 8 9)	22.97	22.97
5	21.82	21.82	(0 1 3)	0.00°	0.00°	(0 2 3)	21.82	21.82
6	21.33	21.33	(0 1 5 7)	0.00°	0.00°	(0 2 6 7)	21.33	21.33
7	20.64	20.64	(0 1 2 3 5)	0.00°	0.00°	(0 2 3 4 5)	20.64	20.64
8	20.42	20.42	(0 1 2 4 6 9)	0.00°	0.00°	(0 2 4 5 6 9)	20.42	20.42
9	20.25	20.25	(0 1 3 7 8)	0.00°	0.00°	(0 1 5 7 8)	20.25	20.25
10	18.85	18.85	(0 1 2 4)	0.00°	0.00°	(0 2 3 4)	18.85	18.85
11	18.06	18.06	(0 1 2 5 7)	0.00°	0.00°	(0 2 5 6 7)	18.06	18.06
12	17.07	17.07	(0 1 2 6 7)	0.00°	0.00°	(0 1 5 6 7)	17.07	17.07
13	16.58	16.58	(0 2 3 4 5 7 9)	0.00°	0.00°	(0 2 4 5 6 7 9)	16.58	16.58
14	15.05	15.05	(0 1 2 3 4 6)	0.00°	0.00°	(0 2 3 4 5 6)	15.05	15.05
15	14.33	14.33	(0 1 2 3 4 5 7)	0.00°	0.00°	(0 2 3 4 5 6 7)	14.33	14.33
16	13.78	13.78	(0 1 3 4 5 6 7 9)	0.00°	0.00°	(0 2 3 4 5 6 8 9)	13.78	13.78
17	13.78	13.78	(0 1 2 3 5 6 7 9)	0.00°	0.00°	(0 2 3 4 6 7 8 9)	13.78	13.78
18	13.78	13.78	(0 1 2 3 4 6 8 9)	0.00°	0.00°	(0 1 3 5 6 7 8 9)	13.78	13.78
19	13.78	13.78	(0 1 2 3 4 6 7 9)	0.00°	0.00°	(0 2 3 5 6 7 8 9)	13.78	13.78
20	13.16	13.16	(0 1 2 3 4 5 6 8)	0.00°	0.00°	(0 2 3 4 5 6 7 8)	13.16	13.16
21	12.25	12.25	(0 1 2 3 4 6 7 8 9)	0.00°	0.00°	(0 1 2 3 5 6 7 8 9)	12.25	12.25
22	12.25	12.25	(0 1 2 3 4 5 7 8 9)	0.00°	0.00°	(0 1 2 4 5 6 7 8 9)	12.25	12.25
23	12.25	12.25	(0 1 2 3 4 5 6 8 9)	0.00°	0.00°	(0 1 3 4 5 6 7 8 9)	12.25	12.25
24	7.50	7.50	(0 1 3 4 7 8)	0.00°	0.00°	(0 1 4 5 7 8)	7.50	7.50
25	6.56	6.56	(0 1 2 4 5 7 8 10)	0.00°	0.00°	(0 1 2 4 6 7 9 10)	6.56	6.56
26	6.56	6.56	(0 1 2 3 5 6 8 10)	0.00°	0.00°	(0 1 2 3 5 7 9 10)	6.56	6.56
27	5.83	5.83	(0 1 2 3 5 6 7 9 10)	0.00°	0.00°	(0 1 2 3 5 6 8 9 10)	5.83	5.83
28	5.83	5.83	(0 1 2 3 4 5 6 7 9)	0.00°	0.00°	(0 2 3 4 5 6 7 8 9)	5.83	5.83
29	3.89	3.89	(0 1 2 3 4 6 7 8 10)	0.00°	0.00°	(0 1 2 3 4 6 8 9 10)	3.89	3.89
30	3.89	3.89	(0 1 2 3 4 5 7 8 10)	0.00°	0.00°	(0 1 2 3 4 5 7 9 10)	3.89	3.89

Table 3: List of the 30 pairs of asymmetrical azimuth-unbiased Tn-types.

Concluding remarks

Historically, the ideas of tonic (major), phonic (minor), and neutral modal genders have been very clear for sets such as the major and minor triads – Tn-types (0 4 7) and (0 3 7), respectively – and the open-fifth – Tn-type (0 5). Nonetheless, this concept of modal gender has proven to be difficult to handle or even useless for other kinds of Tn-types in the context of post-tonal theory. The theoretical approach here proposed specifically allows the extension of this modal grammatical gender concept to sets that are foreign to tonality as traditionally conceived, and such an approach may be able to give interesting and renewed insight into matters such as the formation and use of tonal centers, of modality and of axes of symmetry, using modal gender as a means of organization and study of the Tn-type universe. These ideas come with further promising ramifications to the study of form, due to the renewed major/minor duality and its accompanying dialectics of opposition, and to the study of orchestration, due to the root/vertex concepts and their properties regarding primeval intervals, which yield well-fused sonorities that can be exploited by clever instrumentation. There is also the interesting correlation of gender with transposition and inversion. The fact that tonic Tn-types invert to equal-intensity phonic ones (and vice versa) further the understanding of the relationship between Tn-types and Tn/TnI-types, as well as the comprehension of the possibilities of structural meaning which can be extracted from that relationship of tonic/phonic duality. More importantly, the modal gender and mathematical models here presented not only extend the idea of gender beyond the confines of traditional tonality, but they do this while helping to explain and confirm the historical importance bestowed to the major and minor triads, which are – curiously enough – the Tn-types with the greatest azimuth value possible. In the same spirit of Edmond Costère's *Loi de l'Attraction Universelle* (COSTÈRE, 1954: 15), this expanded concept of modal gender allows for an interesting bridge between tonal and post-tonal music theories and practices, which can be further exploited. Also, the formal theoretical separation between the concepts of root and generatrix, already suggested earlier by ideas such as Ernst Levy's concept of telluric gravity (LEVY, 1985: 15), can be used as an important key element in the reconciliation between nineteenth-century dualism, contemporary psychoacoustics and, more importantly, common musical practice and intuition. In a way, dualist music theories still tend to be elegant and attractive models for musical thought, especially because of their symmetrical conception and their dialectics of modal opposition.

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